5.1 Probability Rules

What is the probability of an event that is impossible? Suppose that a probability is approximated to be zero based on empirical results. Does this mean that the event is impossible?

What is the probability of an event that is impossible?

0 (Type an integer or a decimal.)

Suppose that a probability is approximated to be zero based on empirical results. Does this mean that the event is impossible?

☐ Yes
☐ No

When a probability is based on an empirical experiment, a probability of zero does not mean that the event cannot occur. The probability of an event E is approximately the number of times event E is observed divided by the number of repetitions of the experiment, as shown below. Just because the event is not observed, does not mean that the event is impossible.

\[ P(E) \approx \frac{\text{relative frequency of } E}{\text{number of trials of experiment}} \]

**True or False:** In a probability model, the sum of the probabilities of all outcomes must equal 1.

Choose the correct answer below.

☐ False
☐ True

Probability is a measure of the likelihood of a random phenomenon or chance behavior.

Choose the correct answer below.

☐ False
☐ True

In probability, a(n) experiment is any process that can be repeated in which the results are uncertain.

In probability, an experiment is any process with uncertain results that can be repeated. The result of any single trial of the experiment is not known ahead of time. However, the results of the experiment over many trials produce regular patterns that enable one to predict with remarkable accuracy.
Is the following a probability model? What do we call the outcome "brown"?

<table>
<thead>
<tr>
<th>Color</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>0.1</td>
</tr>
<tr>
<td>green</td>
<td>0.2</td>
</tr>
<tr>
<td>blue</td>
<td>0.1</td>
</tr>
<tr>
<td>brown</td>
<td>0</td>
</tr>
<tr>
<td>yellow</td>
<td>0.2</td>
</tr>
<tr>
<td>orange</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Is the table above an example of a probability model?

A. No, because the probabilities do not sum to 1.

What do we call the outcome "brown"?

A. Not so unusual event
B. Impossible event

Why is the following not a probability model?

Click the icon to view the data table.

<table>
<thead>
<tr>
<th>Color</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>0.2</td>
</tr>
<tr>
<td>Green</td>
<td>-0.2</td>
</tr>
<tr>
<td>Blue</td>
<td>0.1</td>
</tr>
<tr>
<td>Brown</td>
<td>0.4</td>
</tr>
<tr>
<td>Yellow</td>
<td>0.3</td>
</tr>
<tr>
<td>Orange</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Determine why it is not a probability model. Choose the correct answer below.

A. This is not a probability model because the sum of the probabilities is not 1.
B. This is not a probability model because at least one probability is greater than 1.
C. This is not a probability model because at least one probability is greater than 0.
D. This is not a probability model because at least one probability is less than 0.

Which of the following numbers could be the probability of an event?

0.33, -0.44, 0.07, 1.14, 1, 0

The numbers that could be a probability of an event are 0.07, 0.33, 1, 0.0 ≤ P ≤ 1
In a certain card game, the probability that a player is dealt a particular hand is 0.45. Explain what this probability means. If you play this card game 100 times, will you be dealt this hand exactly 45 times? Why or why not?

Choose the correct answer below.

- **A.** The probability 0.45 means that approximately 45 out of every 100 dealt hands will be that particular hand. No, you will not be dealt this hand exactly 45 times since the probability refers to what is expected in the long-term, not short-term.

- **B.** The probability 0.45 means that exactly 45 out of every 100 dealt hands will be that particular hand. Yes, you will be dealt this hand exactly 45 times since the probability refers to short-term behavior, not long-term.

- **C.** The probability 0.45 means that approximately 45 out of every 100 dealt hands will be that particular hand. No, you will not be dealt this hand exactly 45 times since the probability refers to what is expected in the short-term, not long-term.

- **D.** The probability 0.45 means that exactly 45 out of every 100 dealt hands will be that particular hand. Yes, you will be dealt this hand exactly 45 times since the probability refers to long-term behavior, not short-term.

According to a certain country's department of education, 42.4% of 3-year-olds are enrolled in day care. What is the probability that a randomly selected 3-year-old is enrolled in day care?

The probability that a randomly selected 3-year-old is enrolled in day care is **0.424**. (Type an integer or a decimal.)

Let the sample space be \( S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \). Suppose the outcomes are equally likely. Compute the probability of the event \( E = \{1, 4, 7, 9\} \).

\[
P(E) = \frac{4}{10} = 0.4
\]

Let the sample space be \( S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \). Suppose the outcomes are equally likely. Compute the probability of the event \( E = \) "an odd number less than 7."

\[
P(E) = \frac{3}{10} = 0.3
\]
A survey of 600 randomly selected high school students determined that 335 play organized sports.

(a) What is the probability that a randomly selected high school student plays organized sports?

(b) Interpret this probability.

(a) The probability that a randomly selected high school student plays organized sports is \( \frac{335}{600} = 0.558 \).

(Round to the nearest thousandth as needed.)

(b) Choose the correct answer below.

(Type a whole number.)

A. If 1,000 high school students were sampled, it would be expected that about 558 of them play organized sports.

B. If 1,000 high school students were sampled, it would be expected that exactly 0 of them play organized sports.

A bag of 100 tulip bulbs purchased from a nursery contains 25 red tulip bulbs, 20 yellow tulip bulbs, and 55 purple tulip bulbs.

(a) What is the probability that a randomly selected tulip bulb is red?

(b) What is the probability that a randomly selected tulip bulb is purple?

(c) Interpret these two probabilities.

(a) The probability that a randomly selected tulip is red is 0.25.

(Type an integer or a decimal. Do not round.)

(b) The probability that a randomly selected tulip bulb is purple is 0.55.

(Type an integer or a decimal. Do not round.)

(c) Select the correct choice below and fill in the answer boxes within your choice.

(Type whole numbers.)

A. If 100 tulip bulbs were sampled with replacement, one would expect exactly 25 of the bulbs to be red and exactly 55 of the bulbs to be purple.

B. If 100 tulip bulbs were sampled with replacement, one would expect about 25 of the bulbs to be red and about 55 of the bulbs to be purple.
In a national survey college students were asked, "How often do you wear a seat belt when riding in a car driven by someone else?" The response frequencies appear in the table to the right. (a) Construct a probability model for seat-belt use by a passenger. (b) Would you consider it unusual to find a college student who never wears a seat belt when riding in a car driven by someone else?

<table>
<thead>
<tr>
<th>Response</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Never</td>
<td>104</td>
</tr>
<tr>
<td>Rarely</td>
<td>321</td>
</tr>
<tr>
<td>Sometimes</td>
<td>586</td>
</tr>
<tr>
<td>Most of the time</td>
<td>1115</td>
</tr>
<tr>
<td>Always</td>
<td>2525</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>4651</strong></td>
</tr>
</tbody>
</table>

(a) Complete the table below.

<table>
<thead>
<tr>
<th>Response</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Never</td>
<td>.022</td>
</tr>
<tr>
<td>Rarely</td>
<td>.069</td>
</tr>
<tr>
<td>Sometimes</td>
<td>.126</td>
</tr>
<tr>
<td>Most of the time</td>
<td>.240</td>
</tr>
<tr>
<td>Always</td>
<td>.543</td>
</tr>
</tbody>
</table>

(Round to the nearest thousandth as needed.)

(b) Would you consider it unusual to find a college student who never wears a seat belt when riding in a car driven by someone else?

- A. Yes, because $P(\text{never}) < 0.05$.
- B. No, because there were 104 people in the survey who said they never wear their seat belt.
- C. No, because the probability of an unusual event is 0.
- D. Yes, because $0.01 < P(\text{never}) < 0.10$.

John, Dominique, and Clarice work for a publishing company. The company wants to send two employees to a statistics conference. To be fair, the company decides that the two individuals who get to attend will have their names randomly drawn from a hat.

(a) Determine the sample space of the experiment. That is, list all possible simple random samples of size $n = 2$.

(b) What is the probability that John and Dominique attend the conference?

(c) What is the probability that Clarice attends the conference?

(d) What is the probability that John stays home?

(a) Choose the correct answer below. Note that each person is represented by the first letter in their name.

- ✔ A. JD, JC, DC
- B. JD, JC, DC, JJ, DD, CC
- C. JD, JC
- D. JD, JC, DC, DJ, CJ, CD

(b) The probability that John and Dominique attend the conference is $\frac{1}{3} = .3$ when both go at same time (Round to one decimal place as needed.)

(c) The probability that Clarice attends the conference is .7. (Round to one decimal place as needed.)

(d) The probability that John stays home is .3. (Round to one decimal place as needed.)
A baseball player hit 53 home runs in a season. Of the 53 home runs, 19 went to right field, 16 went to right center field, 8 went to center field, 9 went to left center field, and 1 went to left field.
(a) What is the probability that a randomly selected home run was hit to right field?
(b) What is the probability that a randomly selected home run was hit to left field?
(c) Was it unusual for this player to hit a home run to left field? Explain.

(a) The probability that a randomly selected home run was hit to right field is $\frac{19}{53} = .358$.
(Round to three decimal places as needed.)

(b) The probability that a randomly selected home run was hit to left field is .019.
(Round to three decimal places as needed.)

(c) Was it unusual for this player to hit a home run to left field?

- A. Yes, because $P(\text{left field}) < 0.5$.
- B. No, because this player hit 1 home runs to left field.
- C. No, because the probability of an unusual event is 0.
- D. Yes, because $P(\text{left field}) < 0.05$.

You suspect a 6-sided die to be loaded and conduct a probability experiment by rolling the die 400 times. The outcome of the experiment is listed in the following table. Do you think the die is loaded? Why?

<table>
<thead>
<tr>
<th>Value of Die</th>
<th>Frequency in original data set</th>
<th>Value of Die</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>41</td>
<td>4</td>
<td>38</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>41</td>
<td>6</td>
<td>124</td>
</tr>
</tbody>
</table>

Do you think the die is loaded?

Yes, because two of the values have a higher probability of occurring than expected under the assumption of equally likely outcomes.

You suspect a 6-sided die to be loaded and conduct a probability experiment by rolling the die 400 times. The outcome of the experiment is listed in the following table. Do you think the die is loaded? Why?

<table>
<thead>
<tr>
<th>Value of Die</th>
<th>Frequency in original data set</th>
<th>Value of Die</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
<td>4</td>
<td>64</td>
</tr>
<tr>
<td>2</td>
<td>66</td>
<td>5</td>
<td>63</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td>6</td>
<td>73</td>
</tr>
</tbody>
</table>

Do you think the die is loaded?

No, because each value has an approximately equal chance of occurring.

In a recent survey, it was found that the median income of families in country A was $57,200. What is the probability that a randomly selected family has an income greater than $57,200?

What is the probability that a randomly selected family has an income greater than $57,200?

.5 (Type an integer or a decimal.)
Explain the Law of Large Numbers. How does this law apply to gambling casinos?

Choose the correct answer below.

☐ A. As the number of repetitions of a probability experiment increases, the proportion with which a certain outcome is observed gets closer to the probability of the outcome. Casinos use the Law of Large Numbers to determine how many players gamble in certain games.

☐ B. As the number of repetitions of a probability experiment increases, the proportion with which a certain outcome is observed gets closer to the probability of the outcome. This applies to casinos because they are able to make a profit in the long run because they have a small statistical advantage in each game.

☐ C. As the number of repetitions of a probability experiment increases, the proportion with which a certain outcome is observed gets closer to 1. This applies to casinos because they are able to make a profit in the long run because they have a small statistical advantage in each game.

☐ D. As the number of repetitions of a probability experiment increases, the proportion with which a certain outcome is observed gets closer to 0. Casinos use the Law of Large Numbers to determine how many players gamble in certain games.

Describe what an unusual event is. Should the same cutoff always be used to identify unusual events? Why or why not?

Choose the correct answer below.

An event is unusual if it has a low probability of occurring. The same cutoff should not always be used to identify unusual events. Selecting a cutoff is subjective and should take into account the consequences of incorrectly identifying an event as unusual.

Describe the difference between classical and empirical probability.

Choose the correct answer below.

The empirical method obtains an approximate empirical probability of an event by conducting a probability experiment. The classical method of computing probabilities does not require that a probability experiment actually be performed. Rather, it relies on counting techniques, and requires equally likely outcomes.

The empirical method obtains an approximate probability of an event by conducting a probability experiment. The probability is approximate because different runs of the probability experiment lead to different outcomes, and, therefore, different estimates of the probability. The classical method of computing probabilities relies on counting techniques, and requires equally likely outcomes. An experiment has equally likely outcomes when each outcome has the same probability of occurring.